Before you flex your superpowers, you have to deal with the kryptonite.
You have to accept it.

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POSTER KRYPTONITE
APPLAUSE FOR OUR EXAMPLES.
POSTER KRYPTONITE

Abstract*
Centrality Measures of Graphs utilizing Continuous Walks in Hilbert Space

Jarod Benowitz¹, David Peak¹, PhD

¹Utah State University, Physics Department, UT 84321, Email: JarodPBenowitz@gmail.com

ABSTRACT

Centrality is most commonly thought of as a measure in which we assign a ranking of the vertices from most important to least important. The importance of a vertex is relative to the underlying process being carried out on the network. This is why there is a diverse amount of centrality measures addressing many such processes. We propose a measure that assigns a ranking in which interference is a property of the underlying process being carried out on the network.

INTRODUCTION

Networks are perhaps one of the most ubiquitous structures in nature. They arise for example in cellular biology connecting genes and proteins, in neuroscience connecting neuronal regions of the brain, in sociology connecting the interactions of people, and recently in quantum computing. The analysis of the underlying topology of these discrete structures has thus gained widespread attention. Likewise, there has been a significant focus on designing measures to assess certain topological features of a network by assigning quantitative values to the nodes. These quantitative values have a subtle interpretation insofar as there are implicit assumptions of the underlying process being carried out on the network.

Borgatti has identified a typology of flow processes with specific trajectories that use trails, geodesics, paths, or walks. In this framework the flow has a specific type of transmission corresponding to some concrete application. Borgatti gives examples such as used goods, currency, infections, and gossip. Suppose we want to model a flow process in which the flow may interfere with itself. This interference may be the result of collisions in the network where oppositely oriented flows may annihilate. How then can we model such a flow? Our proposition is to model continuous walks on the network insofar as interference becomes an emergent property.

Definition: Centrality is a measure in which the nodes of a network are assigned a ranking with respect to an implicit assumption of the flow characteristics of the network. Below we give several examples of common centrality measures.

Degree Centrality: \[ \text{deg}(i) = \sum_{j=1}^{n} a_{ij} = (Ae)_i \]

Katz Centrality: \[ k(i) = \sum_{j=1}^{n} \alpha(\beta A)^k a_{ij} \]

Closest Centrality: \[ C(i) = \left[ \frac{1}{\beta} d(i, i') \right]^{-1} \]

THEORY

\[ A^t = PD^{t-2} \sum_{i=1}^{N} \lambda_i^{t-1} u_i u_i^t + \sum_{i=1}^{N} \lambda_i^{t-1} u_i u_i^t \]

and where \((-1)^t = e^{\pi i / 2}\). We then can express every entry of \(A^t\) as,

\[ \varphi_{ij}(x_i) = \sum_{k=1}^{N} \lambda_i^{k-1} u_i^k + e^{\pi i x} \sum_{k=1}^{N} \lambda_i^{k-1} u_i^k \]

where \(\lambda^\wedge(0)\) is the multiset of all positive eigenvalues not including zero and \(\lambda^\wedge\) is the multiset of all negative eigenvalues. Since we are guaranteed at least one negative eigenvalue \(\varphi_{ij}(x_i)\) is complex always.\[ \]

Theorem 1. The Pairwise Walk Function (PWF), \(\varphi_{ij}\), is an element of Hilbert Space.

Proof: \[ \int \varphi_{ij}(x_i) \varphi_{jk}(x_j) dx \]

On the right-hand side of the integral we have two indeterminates of the form \(\frac{d}{dx}\) when when \(\lambda_a \to 1\) and when \(\lambda_{b} \to 0\). Upon a change of variable the limit is,

\[ \lim_{\lambda_{a} \to 1} = \frac{d}{dx} = 1 \]

\[ \lim_{\lambda_{b} \to 0} = e^{\pi i x} = 0 \]

The integral then converges over the interval and we have the desired result, \(\varphi_{ij} \in \mathcal{H}\). Below we plot the real and imaginary parts of several PWF’s.

RESULTS

Using the previous theorem we may now define a unique class of centrality measures that live in Hilbert Space. Moreover, we may generalize common centrality measures to account for the additional property of flow self-interference. Below we give Degree Interference and Closeness-Interference, where \(C\) is the sum of the columns of the PWF matrix.

\[ \lambda_{ij} = \sum_{k=1}^{N} A_k^{ij} \]

\[ \sum_{k=1}^{N} A_k^{ij} \sum_{k=1}^{N} A_k^{jk} \]

Figure 3. An inverse relationship between Closeness and Closeness-Interference. Closeness-Interference starts the peripheral vertices closer than the core vertices. We may attribute this to destructive interference among the core vertices.

CONCLUSION

We’ve shown that when we allow continuous processes to occur on discrete structures interference becomes an emergent property. In this manner we may view graphs as lower-dimensional discrete representations of Hilbert space. To the authors knowledge this is the first explicit relationship between combinatorics and Hilbert space. Using this to our advantage we’ve generalized several common centrality measures to account for flow self-interference. Furthermore, these measures may be used for the development of new and novel quantum algorithms. Likewise, we saw an interesting relationship between numerical simulations of quantum random walks in 1D with the PWF for the path graph. Keeping the Distance Minimizer theorem in mind, which states that for all vectors in Hilbert space there exists a unique vector in a closed subspace of Hilbert space, which minimizes their distance, we may utilize PWFs as approximations to quantum random walks. Finally, an intriguing prospect is whether or not we can construct linear Hermitian operators corresponding to graph parameters just as we have linear Hermitian operators that correspond to physical observables in quantum mechanics.

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Katz Centrality: 

\[ k(i) = \sum_{j=1}^{n} a_{ij} (\sum_{k=1}^{n} a_{jk})^{-(s-1)} \]

Closest Centrality: 

\[ c(i) = \left[ \frac{\sum_{j=1}^{n} d(i,j)}{n} \right]^{-1} \]

THEORY

\[ A^k = PD^{-1}F^{-1} \left( \sum_{i=1}^{n} \lambda_i^{-(k-1)} u_i u_i^T + \varepsilon e e^T + \sum_{j=1}^{n} \lambda_j^{-(k-1)} u_j u_j^T \right) \]

and where \((-1)^{k(\bar{r})}\lambda_i^{r} \text{exp}(\varepsilon)\). We then can express every entry of \( A^k \) as, 

\[ \varphi_{jk}(x,k) = \sum_{i=1}^{n} \lambda_i^{k} u_i^k + \varepsilon \sum_{i=1}^{n} \lambda_i^{k} u_j^k \]

where \( \lambda_i \) is the multiset of all positive eigenvalues not including zero and \( \lambda_i \) is the multiset of all negative eigenvalues. Since we are guaranteed at least one negative eigenvalue, \( \varphi_{jk}(x,k) \) is complex always \( \sqrt{\lambda_i} \).

Theorem 1: The Pairwise Walk Function (PWF), \( \varphi_{jk} \), is an element of Hilbert Space.

Proof:

\[ \int (\sin x) \varphi_{jk}(x,k) dx = \int (\sin x) \left( \sum_{i=1}^{n} \lambda_i^{k} u_i^k + \varepsilon \sum_{i=1}^{n} \lambda_i^{k} u_j^k \right) dx \]

On the right-hand side of the integral we have two independently bounded terms with when \( x \rightarrow \pi \), \( k \rightarrow \infty \). Upon a change of variable the limit is,

\[ \frac{\sin x}{x} \rightarrow 1 \text{ as } x \rightarrow 0 \]

The integral then converges over the interval and we have the desired result, \( \varphi_{jk} \in \mathcal{H} \). Below we plot the real and imaginary parts of several PWF’s.

RESULTS

Using the previous theorem we may now define a unique class of centrality measures that live in Hilbert Space. Moreover, we may generalize common centrality measures to account for the additional property of flow self-interference. Below we give Degree-Interference and Closeness-Interference, where \( \mathcal{C} \) is the sum of the columns of the PWF matrix.

\[ \mathcal{C} = \left[ \int \left( \sum_{i=1}^{n} \lambda_i^{k} u_i^k + \varepsilon \sum_{i=1}^{n} \lambda_i^{k} u_j^k \right) dx \right] \]

We’ve shown that when we allow continuous processes to occur on discrete structures interference becomes an emergent property in this manner we way view graphs as lower-dimensional discrete representations of hilbert space. To the authors knowledge this is the first explicit relationship between combinatorics and hilbert space. Using this to our advantage we’ve generalized several common centrality measures for account for flow self-interference. Furthermore, these measures may be used for the development of new and novel quantum algorithms. Likewise, we saw an interesting relationship between numerical simulations of quantum random walks in 1D with the PWF for the path graph. Keeping the Distance Minimizer theorem in mind, which states that for all vectors in hilbert space there exists a unique vector in a closed subspace of hilbert space, which minimizes their distance, we may utilize PWF’s as approximations to quantum random walks. Finally, an intriguing prospect is whether or not we can construct linear hermitian operators corresponding to graph parameters just as we have linear hermitian operators that correspond to physical observables in quantum mechanics.

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Katz Centrality:
\[ k(i) = \sum_{j=1}^{n} a^{ij}(A^T)^i_j = \left( (I - A)I^T \right)^{-1} \]

Closeness Centrality:
\[ c(i) = \frac{1}{\sum_{j=1}^{n} d(i,j)} \]

THEORY

\( A^\prime = PD^{-1}P^T \quad \text{and where } (-)^\prime = L^T(-L) \).

Theorem 1: The Pairwise Walk Function (PWF), \( \varphi_{jk} \), is an element of Hilbert Space.

On the right-hand side of the integral we have two indeterminates of the form \( \mathcal{H} \) when when \( \mu_n \rightarrow 1 \) and when \( \mu_n \lambda_{i=k} \rightarrow 1 \). Upon a change of variable the limit is

\[ \int_0^1 \left( e^{\mu_n \lambda_{i=k}} - 1 \right) \frac{d\mu}{\lambda_{i=k}} \]
POSTER KRYPTONITE

Abstract*
Background images
Time-of-Flight Mass Spectrometry (TOFMS) is a technique for determining particle mass using a temporal data spectrum. Charged particles are accelerated through an electric potential, with higher resulting particle speeds corresponding to particles with lower mass. A particle's time of arrival is measured and used to determine the particle mass.

**Mission:** Data from density and composition studies of Earth's upper atmosphere are used to improve atmospheric models. The Miniaturized Time-of-Flight Mass Spectrometer will be designed for a CubeSat bus and will be capable of providing data with better temporal and spatial resolution than previous instruments flown on larger satellites. This design aims to leverage full-scale TOF resolution techniques to achieve mass resolution comparable to larger instruments.

**Search for Optimal Dimensions:** Optimization functions written in MATLAB calculated maximum drift region lengths given a set of dimensions [reflectron depth, spacing between accelerators, accelerator voltages], calculated flight times for 60 AMU, and evaluated each dimension set based on a spacing parameter.

\[
\text{spacing} = \frac{\text{width of 60 AMU peak [seconds]}}{\text{distance between 59 and 60 AMU peaks [seconds]}}
\]

Outcomes of the dimension search suggested larger dimensions for the reflectron depth [55 mm reflectron design pictured].

**BNG Driver Design:** Alternating wires of the BNG may be driven using a high-speed high-side/low-side boost driver and high voltage, high speed MOSFET switches. Electrical parameters from a previously fabricated BNG were used to simulate the BNG and evaluate the driver performance.

**SPICE simulations of the BNG driver show ion pulse widths less than 35 nanoseconds. Power consumption will be evaluated and further improvements in rise time and pulse width may be possible.**

**MCP Signal Collector Design:** Storage of data from a Constant Fraction Discriminator (CFD) or Analog-to-Digital Converter (ADC) in a high speed register to be transferred at larger intervals to an onboard computer will balance timing requirements for signal sampling and power consumption of onboard computers.

**Flight time estimations and SIMION simulation results show similar resolving power. Flight time estimation was run using a 30 nanosecond Time of Birth (TOB) range. SIMION simulations were run for a 30 nanosecond TOB range and a 0.42 mm starting position range (based on a 50 nanosecond gate pulse and thermal velocity distribution of the particles).**

**SIMION was used to evaluate reflectron electrode potential sets and electrode shapes.**
Abstract*

Background images

Flanking logos
Background
• Approximately 10 million tons of Waste Vegetable Oils are generated annually in the United States of America alone [1].
• Waste Vegetable Oils (WVO) are potential sources for fuels and hydrogen gas production [2].
• Hypothesis: Pyrolysis of WVOs using red mud produces hydrogen and low oxygenated fuels.

Importance
• Dual production of hydrocarbons and hydrogen.
• Reutilization of waste materials.

Chemistry of WVO
Triglyceride \(\rightarrow\) Fatty acids + Glycerol

Objectives
• Investigate the effect of red mud on the pyrolysis of model compounds.
• Study the influence of red mud on the pyrolysis of WVO.
• Determine the optimum conditions for the production of hydrogen and hydrocarbons are produced from WVO.

Materials
• Model compounds: Glycerol, oleic acid, linoleic acid, trilaurin and triolein.
• Extraction solvents: Tetrahydrofuran and hexane.
• Catalyst: Red mud.
• Waste vegetable oil.

Expected Results
• Reaction mechanisms of pyrolytic products using red mud catalyst.

Triglyceride model

Method
• Non-catalytic pyrolysis of model compounds and at 390°C, 420°C and 450°C.
• Catalytic pyrolysis at different percentages.
• Analysis of pyrolytic products.

Conclusions
• Pyrolysis of WVO will reduce dependence on fossil fuels.
• Production of H2 – an essential industrial commodity.

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Katz Centrality: \( k(i) = \sum_{j=1}^{n} \sum_{k=1}^{n} k(i,j) \)

Closest Centrality: \( C(i) = \left[ \sum_{j=1}^{n} d(i,j)^{-1} \right]^{-1} \)

THEORY

\( A^2 = P D P^{-1} + \sum_{k=1}^{n} A_k A_k \)

where \( k(i,j) \), \( \lambda_k \), and \( A_k \) are the eigenvalues and eigenvectors of \( A \). We then can express every entry of \( A^2 \) as,

\[ \rho(i,j) = \sum_{k=1}^{n} \lambda_k A_k \]

where \( \lambda_k \) is the sum of all positive eigenvalues not including zero and \( \lambda_k \) is the multiset of all negative eigenvalues. Since we are guaranteed at least one negative eigenvalue \( \rho(i,j) \) is complex always.

Theorem 1: The Pairwise Work Function (PWF), \( \rho(i,j) \), is an element of Hilbert Space.

Proof:

On the right-hand side of the integral we have two indeterminates of the form \( \sum_{k=1}^{n} \).

The integral then converges over the interval and we have the desired result, \( \rho(i,j) \in \mathcal{H} \).

Below we plot the real and imaginary parts of several PWF's.

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Using the previous theorem we may now define a unique class of centrality measures that live in Hilbert Space. Moreover, we may generalize common centrality measures to account for the additional property of flow self-interference. We give Degree Interference and Closeness Interference, where \( i \) is the sum of the columns of the PWF matrix.

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\[ k(i) = \sum_{j=1}^{N} a^{ij}(A^j) = \left( (I - (1-\eta)A)^{-1} - I \right) e_i \]

Closeness Centrality:

\[ c(i) = \frac{1}{\sum_{j=1}^{N} d(i,j)} \]

Theorem 1.1: The power-law function \( f(i) = i^\alpha \), \( \alpha \in \mathbb{R} \), is the adjacency matrix produces complex functions as the entries.

Proof: Let \( A \) be the adjacency matrix of a simple nonempty graph. \( A \) is a traceless symmetric matrix, \( \text{Tr}[A] = \text{Tr}[A^T] = 0 \). Since \( A \) is symmetric it is always diagonalizable, we then have \( A = (P\text{P}^T)^{-1} \) where \( P \) are the eigenvectors collected as a matrix and \( D \) is the diagonal matrix consisting of the eigenvalues of \( A \). We then have \( \text{Tr}[A] = \text{Tr}[(P\text{P}^T)^{-1}] = \text{Tr}[(D\text{D}^T)^{-1}] = \text{Tr}(D) = \sum_{i=1}^{N} \lambda_i = 0 \). Since the graph is nonempty and the sum of the eigenvalues is zero we are therefore guaranteed to have at least one negative eigenvalue. The function of a matrix can be expressed as \( f(A) = P(D^a)P^T \), where the spectral decomposition is.

THEORY

\[ A^2 = PD^2P^{-1} - \sum_{i,a} n_i a_i P_{ia} = \sum_{i,a} n_i a_i P_{ia} \]

and where \((-\lambda_i)^a = \lambda_i e^{i2\pi a}\). We then can express every entry of \( A^2 \) as,

\[ \varphi_{jk}(l_1,l_2) = \sum_{\mathcal{L}} \lambda_i^{l_1} u_{jL} + v_{kl} \sum_{\mathcal{L}} \lambda_i^{l_2} u_{kL} \]

where \( \lambda^\mathcal{L}_i(\mathcal{L}) \) is the multiset of all positive eigenvalues not including zero and \( \lambda^-\mathcal{L}_i \) is the multiset of all negative eigenvalues. Since we are guaranteed at least one negative eigenvalue \( \varphi_{jk}(l_1,l_2) \) is complex always.

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\[ D_{ij} = \left( \sum_{\mathcal{L}} \mathcal{L}_i \mathcal{L}_j \right)^{-1} \]

\[ C_{ij} = \left( \sum_{\mathcal{L}} \mathcal{L}_i \mathcal{L}_j \right)^{-1} \]

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A Study on the Effect between Commercial Space Solar Cells and the Antennas Integrated on Their Cover Glass

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Abstract

A study to determine how commercial space solar cells affect the functionality of the antenna integrated on top of solar cells has been performed. The measured results show that solar cell affects the antenna gain and decreases it by approximately 3 dB at 5GHz. In addition, the pattern of the antenna was not affected significantly by solar cells whether when they were illuminated and terminated with different loads.

Introduction

One of the biggest issues of cube satellites is the limited surface area, that makes it challenging to place antenna on cube satellites without competing for the surface area with solar cells. One effective method can be integrate antenna with solar cells.

In the past, two types of this kind of integration has been performed at Utah State University.

As antennas integrated on top of solar cells offers lots of advantages, it is important to determine the effect of solar cells on the antenna.

Fabrication

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Plexiglass Cover Glass Results:

| G_{no_solar} = 5 dBi |
| G_{solar} = 2.4 dBi |

Conclusion

Solar cells decrease 3 dB antenna gain without disturbing radiation pattern. Also, solar cells affect impedance of feedline and antenna. Finally, the gain of patch antenna on solar cell is independent of solar cell loading and activeness.

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Department of Electrical and Computer Engineering, Utah State University, Logan, UT 84322, USA
taha.shahvirdi@aggiemail.usu.edu, reyhan.baktur@usu.edu

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Table 1- A simple way to display numbers and figures

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II. Methods

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Abstract*
Background images
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A Study on the Effect between Commercial Space Solar Cells and the Antennas Integrated on Their Cover Glass

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Abstract*
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Flanking logos
Mailing addresses
Drop shadows and bevels
Centrality Measures of Graphs utilizing Continuous Walks in Hilbert Space

Jarod Benowitz¹, David Peak¹, PhD
¹Utah State University, Physics Department, UT 84321, Email: JarodPBenowitz@Gmail.com

ABSTRACT

Centrality is most commonly thought of as a measure in which we assign a ranking of the vertices from most important to least important. The importance of a vertex is relative to the underlying process being carried out on the network. This is why there is a diverse amount of centrality measures addressing many such processes. We propose a measure that assigns a ranking in which interference is a property of the underlying process being carried out on the network.

THEORY

\[ A^* = P(D^{-1})A, \]

and where \((-X)^+ = \lambda^* X^{+\lambda^*} \). We then can express every entry of \( A^* \) as,

\[ \varphi_{ij}(x) = \sum_{i,j \in \mathbb{N}_0} \lambda_i^j u_{ij} + \phi^{\text{zeros}} \sum_{i,j \in \mathbb{N}_0} \lambda_i^j u_{ij} \]

where \(\lambda^\gamma \alpha \) is the multiset of all positive eigenvalues not including zero and \(\lambda^* \) is the multiset of all negative eigenvalues. Since we are guaranteed at least one negative eigenvalue \(\varphi_{ij}(x) \) is complex always.

Theorem 1. The Pairwise Walk Function (PWF), \(\varphi_{ij} \), is an element of Hilbert Space.

Proof: \[
\sum_{i,j \in \mathbb{N}_0} \lambda_i^j u_{ij}\]

On the right-hand side of the integral we have two indeterminates of the form \(\frac{1}{x} \) when \(\lambda i \to 1 \) and \(\lambda j \to 1 \). Upon a change of variable the limit is,

\[ \lim_{x \to 1} \frac{\ln(\lambda i) - 1}{\ln(\lambda j) - 1} = L \]

The integral then converges over the interval and we have the desired result, \(\varphi_{ij} \in \mathcal{H} \). Below we plot the real and imaginary parts of several PWF’s.

RESULTS

Using the previous theorem we may now define a unique class of centrality measures that live in Hilbert Space. Moreover, we may generalize common centrality measures to account for the additional property of flow self-interference. Below we give Degree-Interference and Closeness-Interference, where \(c \) is the sum of the columns of the PWF matrix.

\[ D_i = \sum_{j \in \mathbb{N}_0} \phi_{ij}(x)^2 \]

\[ C_i = \sum_{i,j \in \mathbb{N}_0} \phi_{ij}(x)^2 \]

Figure 3: An inverse relationship between Closeness and Closeness-Interference. Closeness-Interference marks the peripheral vertices closer than the core vertices. We may attribute this to destructive interference among the core vertices.

CONCLUSION

We’ve shown that when we allow continuous processes to occur on discrete structures interference becomes an emergent property. In this manner we may view graphs as lower-dimensional discrete representations of Hilbert space. To the authors knowledge this is the first explicit relationship between combinitorics and Hilbert space. Using this to our advantage we’ve generalized several common centrality measures to account for flow self-interference. Furthermore, these measures may be used for the development of new and novel quantum algorithms. Likewise, we saw an interesting relationship between numerical simulations of random walks on 1D with the PWF for the path graph. Keeping the Distance Minimizer theorem in mind, which states that for all vectors in Hilbert space there exists a unique vector in a closest subspace of Hilbert space, which minimizes their distance, we may utilize PWFs as approximations to quantum random walks. Finally, an intriguing prospect is whether or not we can construct linear hermitian operators corresponding to graph parameters just as we have linear hermitian operators that correspond to physical observables in quantum mechanics.

ACKNOWLEDGEMENTS

I thank Dr. David Brown for his constructive criticism and referee report of the paper. I also thank the Fall 2014 Graph Theory class for their constructive criticism.

REFERENCES


Centrality Measures of Graphs utilizing Continuous Walks in Hilbert Space

Jarod Benowitz\textsuperscript{1}, David Peak\textsuperscript{1}, PhD

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ABSTRACT
Centrality is most commonly thought of as a measure in which we assign a ranking of the vertices from most important to least important. The importance of a vertex is relative to the underlying process being carried out on the network. This is why there is a diverse amount of centrality measures addressing many such processes. We propose a measure that assigns a ranking to which interference is a property of the underlying process being carried out on the network.

INTRODUCTION
Networks are perhaps one of the most ubiquitous structures in nature. They arise for example in cellular biology connecting genes and proteins, in neuroscience connecting neurological regions of the brain, in sociology connecting the interactions of people, and recently in quantum computing. The analysis of the underlying topology of these discrete structures has thus gained widespread attention. Likewise, there has been a significant focus on designing measures to assess certain topological features of a network by assigning quantitative values to the nodes. These quantitative values have a subtle interpretation insofar as they are implicit assumptions of the underlying process being carried out on the network.

Borgatti has identified a typology of flow processes with specific trajectories that use trails, geodesics, paths, or walks. In this framework the flow has a specific type of transmission corresponding to some concrete application. Borgatti gives examples such as used goods, currency, infections, and gossip. Suppose we want to model a flow process in which the flow may interfere with itself. This interference may be the result of collisions in the network where oppositely oriented flows may annihilate. How then can we model such a flow? Our proposition is to model continuous walks on the network insofar as interference becomes an emergent property.

Definition: Centrality is a measure in which the nodes of a network are assigned a ranking with respect to an implicit assumption of the flow characteristics of the network. Below we give several examples of common-centrality measures.

Degree Centrality: \[
\text{deg}(i) = \sum_{j=1}^{N} a_{ij} = (\mathcal{A}e)_i
\]

Katz Centrality: \[
k(i) = \sum_{j=1}^{N} a^{k-1}(\mathcal{A}^k)_j = \left((I-\alpha\mathcal{A})^{-1} - I\right)e)_i
\]

Closeness Centrality: \[
c(i) = \frac{\sum_i}{\sum_j d(i,j)}^{-1}
\]

THEORY

Lemma 1.1: The power-law function \( f(x) = x^\epsilon, \epsilon \in \mathbb{R} \) of the adjacency matrix produces complex functions as the entries.

Proof: Let \( \mathcal{A} \) be the adjacency matrix of a simple nonempty graph. \( \mathcal{A} \) is a traceless symmetric matrix, \( \text{Tr}[\mathcal{A}] = \langle \mathcal{A}, \mathcal{A} \rangle = 0 \). Since \( \mathcal{A} \) is symmetric it is always diagonalizable, we then have \( \mathcal{A} = P\mathbf{\Lambda}P^{-1} \) where \( P \) are the eigenvectors collected as a matrix and \( \mathbf{\Lambda} \) is the diagonal matrix consisting of the eigenvalues of \( \mathcal{A} \). We then have \( \text{Tr}[\mathcal{A}] = \text{Tr}[P\mathbf{\Lambda}P^{-1}] = \text{Tr}(\mathbf{\Lambda}) = \sum \lambda_j = 0 \). Since the graph is nonempty and the sum of the eigenvalues is zero we are therefore guaranteed to have at least one negative eigenvalue. The function of a matrix can be expressed as \( f(\mathcal{A}) = P^{-1}f(\mathbf{\Lambda})P \), where the spectral decomposition is.

\[ A^\epsilon = P\mathbf{\Lambda}^\epsilon P^{-1} - \sum_{k=1}^N \lambda_i^\epsilon u_i u_j = \sum_{k=1}^N \lambda_i^\epsilon u_k \]

where \((-\lambda)^\epsilon = \lambda^\epsilon e^{i\epsilon\pi/2}\). We then can express every entry of \( A^\epsilon \) as,

\[ \varphi_{jk}(\epsilon, x) = \sum_{\epsilon \in \mathbb{R}} \lambda_i^\epsilon u_i + e^{i\epsilon\pi/2} \sum_{\epsilon \in \mathbb{R}} \lambda_j^\epsilon u_j \]

Theorem 1: The Pairwise Walk Function (PWF), \( \varphi_{jk} \), is an element of Hilbert Space.

Proof: \[
\int \varphi_{jk}(\epsilon, x) \varphi_{jk}(\epsilon', x) dx = \int \left( \sum_{\epsilon \in \mathbb{R}} \lambda_i^\epsilon u_i + e^{i\epsilon\pi/2} \sum_{\epsilon \in \mathbb{R}} \lambda_j^\epsilon u_j \right) \left( \sum_{\epsilon' \in \mathbb{R}} \lambda_i^{\epsilon'} u_i + e^{i\epsilon'\pi/2} \sum_{\epsilon' \in \mathbb{R}} \lambda_j^{\epsilon'} u_j \right) dx
\]

On the right-hand side of the integral we have two indeterminates of the form \( i^p \) when when \( \lambda_i \to 1 \) and when \( \lambda_j \to 1 \). Upon a change of variable the limit is,

\[
\lim_{\lambda_j \to 1} \frac{a^\epsilon - 1}{\ln(\lambda_j)} = \frac{a^\epsilon - 1}{\ln(1)} \leq 0
\]

The integral then converges over the interval and we have the desired result, \( \varphi_{jk} \in \mathcal{H}_\epsilon \). Below we plot the real and imaginary parts of several PWF’s.

RESULTS

Using the previous theorem we may now define a unique class of centrality measures that live in Hilbert Space. Moreover, we may generalize common centrality measures to account for the additional property of flow self-interference. Below we give Degree, Interference and Closeness-Interference, where \( \epsilon \) is the sum of the columns of the PWF matrix.

\[ D_{ij} = \int \left| \varphi_{ij}(\epsilon, x) \right|^2 dx = \left( \sum_{\epsilon \in \mathbb{R}} \lambda_i^\epsilon u_i + e^{i\epsilon\pi/2} \sum_{\epsilon \in \mathbb{R}} \lambda_j^\epsilon u_j \right)^2 \]

\[ C_{ij} = \left| \sum_{\epsilon \in \mathbb{R}} \int \varphi_{ij}(\epsilon, x) \varphi_{ij}(\epsilon, x) dx \right| \]

Figure 1. An inverse relationship between Closeness and Closeness-Interference. Closeness-Interference out the peripheral vertices closer than the core vertices. We may attribute this to destructive interference among the core vertices.

CONCLUSION

We’ve shown that when we allow continuous processes to occur on discrete structures interference becomes an emergent property. In this manner we may view graphs as lower-dimensional discrete representations of hilbert space. To the authors knowledge this is the first explicit relationship between combinactorics and hilbert space. Using this to our advantage we’ve generalized several common centrality measures to account for flow self-interference. Furthermore, these measures may be used for the development of new and novel quantum algorithms. Likewise, we saw an interesting relationship between numerical simulations of quantum random walks in 1D with the PWF for the path graph. Keeping the Distance Minimizer theorem in mind, which states that for all vectors in hilbert space there exists a unique vector in a closed subspace of hilbert space, which minimizes their distance, we may utilize PWFs as approximations to quantum random walks. Finally, an intriguing prospect is whether or not we can construct linear hermitian operators corresponding to graph parameters just as we have linear hermitian operators that correspond to physical observables in quantum mechanics.

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REFERENCES
Centrality Measures
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Space
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Abstract*

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